Complete the square. $\left(\frac{b}{2a}\right)^2$

$ax^2 + bx + c = 0$ $\quad \frac{-4ac}{4a^2}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

6-5 The Quadratic Formula and the Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex 1

$3x^2 + x - 1 = 0$ $\quad a=3 \quad b=1 \quad c=-1$

$$x = \frac{-1 \pm \sqrt{1 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-1 \pm \sqrt{13}}{6}$$

ex 2

$5x^2 + 8 = -12x$

$$5x^2 + 12x + 8 = 0$$

$a=5$
$b=12$
$c=8$

$$x = \frac{-12 \pm \sqrt{144 - 4(5)(8)}}{10}$$

$$\frac{-12 \pm 4i}{10}$$

$$x = \frac{-6 \pm 2i}{5}$$

The Discriminant

$$D = b^2 - 4ac$$

Determines the nature of the roots.

Three Cases

I. $D > 0$ 2 Real roots

II. $D = 0$ Double R root

III. $D < 0$ 2 imaginary roots

Ex

$x^2 - 8x + 5 = 0$

Determine the nature of roots

$D = b^2 - 4ac$

$64 - 4(1)(5)$

$44$

2 Real irrational roots

Also able to determine if the roots are rational or irrational.

Rational

a, b, & c must be rational and D must be a perfect square (Real)

ex

$x^2 + 10x + 25 = 0$

$D = 100 - 4(25) = 0$

Double R, rational

ex

$x^2 - 4x + 13 = 0$

$D = 16 - 4(1)(13)$ 2 imaginary

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Determine as much as you can about the roots:

B 1. $y^2 - 3y - 1 = 0$ 2 R irr.

2. $3a^2 - 10a = -11$ 2 imaginary

$-$

0 3. $5x^2 + 2\sqrt{10}x + 2 = 0$ 2 R double R irration

4. $3b^2 = 14b + 24$

Find the value for k such that there are 2 imaginary roots

ex

$5x^2 - 2x + k = 0$

$D < 0$ $4 - 4(5)k < 0$

$4 < 20k$

$\frac{1}{5} < k$

Find the value for k such that there is a double root

ex

$3x^2 + 2x + k = 0$

$D = 0$ $4 - 4(3)k = 0$

$4 = 12k$

$1/3 = k$

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